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INTERNAL WAVE GENERATION BY SUBMERGED BODIES: MEAN FLOW EFFECTS

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13. ABSTRACT

The basic physical processes which are associated with the passage of a body thru a density stratified environment and which result in disturbance motions which are steady in a reference frame moving with the body are systematically investigated. A comparison of the relative internal wave making efficiency of the various mechanisms is made, based on the internal wave energy production rates associated with the mechanisms as functions of the sensible parameters of the problem.

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CONTENTS

Internal Wave Generation by Submerged Bodies: Mean Flow Effects

Introduction

The passage of a body thru a density stratified fluid presents a complex localized disturbance to the system. It is a general property of such systems that disturbance energy is propagated away from the source region in the form of an internal gravity wave field. However, in many circumstances, the complexity of the disturbance created by a real body moving within a real fluid effectively destroys the possibility of an exact mathematical description of the radiated wave field, and we must be content with an attempt to distill the essential physics of the phenomenon. That is, we must isolate and understand those mechanisms which characterize the disturbance and its associated wave field. particularly those mechanisms which transcend individual peculiarities of particular bodies and environments, in the hope that the physical picture which emerges will provide a useful and viable description of the real phenomenon.

On a primitive level, the disturbance may be resolved into three categories on the basis of temporal dependence. In nature we must deal with real fluids, so that for physically interesting situations the disturbance will be characterized by turbulence arising from boundary layer instability. The essence of turbulent motion is randomness, and all aspects of the phenomenon which require a stochastic description will be designated as "random effects". The remainder of the motions associated with the disturbance and the radiated wave field may be designated as steady or "mean field effects" or unsteady or "transient effects". If the body is in steady horizontal motion over a flat bottom, then in a reference frame moving with the body, all non-random effects will be mean field effects. In such a frame of reference, transient effects arise only as a result of unsteadiness in the motion of the body or apparent unsteadiness associated with the boundary configuration.

The mean field effects associated with a submerged body admit a three-fold classification. The first effect is that which would be created by the body in the complete absence of viscous effects. The disturbance is associated with the fact that the body must systematically displace and replace fluid as it moves, and will be referred to as the "hull effect" in the sequel. The second effect, the so-called "wake collapse effect", arises because, in most physically interesting situations, a turbulent wake forms in the lee of the body. Mixing within the

wake results in an effective perturbation of the ambient density field, which brings into play relatively strong buoyancy forces tending to destroy the density perturbation by flattening or collapsing the wake region. The presence of a turbulent wake also presents the possibility of an effective "displacement body" associated with streamline displacements outside of the turbulent region arising from momentum redistribution within the wake region. This third effect, the "displacement thickness effect", is distinct from the wake collapse phenomenon, being similar in its internal wave making characteristics to the hull effect, although its interaction with the ambient density field may result in the distortion of the turbulent wake structure.

All of the mechanisms mentioned above are capable of affecting the radiated internal wave field in some way, the exact details of the wave field depending to some extent on the exact details of the body geometry, unsteady motions and the ambient stratification. Because of the uncertainty involved, exact comparisons of calculated waveforms are of limited utility; in order to assess the relative importance of the various contributing causes, their effects must be characterized by some global property which is somewhat insensitive to the exact details of the flow structure. For this reason, we choose the rate of internal wave energy production as a convenient index of the source strength, and explicit internal wave fields are not computed. It is hoped that the comparison, based on this global property, of the relative importance of the various contributing mechanisms as a function of the sensible parameters which characterize the system, will contribute to the basic understanding of the physics of the phenomena associated with the passage of a body thru a density stratified fluid.

<u>The hull effect</u>

The passage of a solid body thru a fluid environment is accompanied by a continuous readjustment of fluid elements. The bow end continuously displaces fluid while fluid must continuously refill the void left by the motion of the tail end, and the problem is then to understand the processes involved in the interaction of this disturbance with the ambient density stratification of the environment. Most of the basic physics of this phenomenon is contained in the standard mountain lee wave problem for which an extensive literature exists (see reviews by Queney et al, 1960, Miles, 1969, Zeytounian, 1969a, b, Vergeiner, 1971, Lilly, 1972; see also Bretherton, 1969). Reducing the phenomenon to its purest form, we consider the steady horizontal motion of a body in an unbounded uniformly stratified Boussinesq fluid*. The problem may be solved by

*In the Boussinesq approximation (see, for example, Phillips, 1966, sec. 2.4) density variations are considered important only insofar as they determine the buoyancy, which is consistent with our program of retaining only the essence of the problem.

following Dorodnitsyn (see Tareyev, 1965) and replacing the body by an equivalent forcing function which stratifies the boundary conditions. Conceptually, the simplest relevant forcing is that provided by a moving dipole, for which solutions have been presented by Crapper (1959), Crosch (1964) and Miles (1971a). The general formalism for the solution of problems of general localized forcing in a stratified flow has been developed by Lighthill (1967) and Liu and Yeh (1971). However, we will approach the problem in the classical mountain lee wave tradition, developing an integral transform solution to the boundary value problem*. As noted previously, there is a vast literature on the classical lee wave problem, and therefore we will give a rather concise derivation of the solution. A thorough discussion of integral transform techniques is given by Sneddon (1972).

Although the full nonlinear problem for a two dimensional object such as a ridge or horizontal cylinder aligned perpendicular to a uniform stratified flow may be solved by invoking Long's model (Long, 1953, 1955, see also Miles, 1969, McIntyre, 1972), the full three dimensional problem remains intractable unless linearized by the assumption that the disturbance is a small perturbation to the basic state. A measure of the relative importance of nonlinear effects is provided by the inverse internal Froude number

$$F^{-1} = \frac{NH}{U} \tag{1}$$

where U is the speed of the oncoming stream (the speed of the body in a stationary frame of reference), H is a characteristic value of the vertical dimension of the body (the maximum radius of a body of revolution), and N is the Brunt-Vaisala frequency

$$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} \tag{2}$$

where g is the acceleration of gravity and $\frac{1}{\rho}\frac{d\rho}{dz}$ is the vertical logarithmic density gradient in the basic state. Conceptually, the parameter F^{-1} is the square root of the ratio of the work required to raise an element of fluid to the top of the obstacle to the kinetic energy density of the oncoming fluid. The availability of the finite amplitude solution for the two dimensional problem permits the evaluation of the validity of the linearized solution as a function of F^{-1} for that problem (see Miles, 1969, 1971b). These two-dimensional results indicate that the linearized problem should provide a reasonable

*A somewhat more general approach to the boundary value problem for an isolated body in a stratified fluid is considered by Mackinnon <u>et al</u> (1969).

approximation for F^{-1} less than 1/4 or 1/2 although, as will be discussed later, some error may be introduced by the linearized boundary conditions for smaller values of F^{-1} for certain body shapes. The approximate three dimensional solution in the limit of large F^{-1} has been discussed by Drazin (1961) and Grimshaw (1969), and that for very small F^{-1} by Hawthorne and Martin (1955) and Drazin (1961).

The linearized momentum balance equations for an inviscid, Boussinesq fluid in a reference frame moving with the body (Fig. 1) are

$$U\frac{\partial u}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \tag{3a}$$

$$U\frac{\partial v}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \tag{3b}$$

$$U\frac{\partial w}{\partial x} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{\rho}{\rho_0}g \tag{3c}$$

where U is the speed of the body, ρ_0 is a reference density, and u, v, w, ρ, p are perturbations of the velocity components, density, and pressure. Mass conservation and incompressibility are expressed respectively by

$$U\frac{\partial \rho}{\partial x} + w\frac{d\bar{\rho}}{dz} = 0 \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{5}$$

where $\bar{\rho}$ is the basic density profile, which is a linear function of the vertical coordinate. Equations (3) - (5) may be combined to form a governing equation for the amplitude function, η,

$$\nabla^2\frac{\partial^2\eta}{\partial x^2} + \frac{N^2}{U^2}\nabla_h^2\eta = 0 \tag{6}$$

where $U\,\partial\eta/\partial x = w$, ∇^2 is the Laplacian operator and ∇_h^2 is the horizontal Laplacian ($\partial^2/\partial x^2+\partial^2/\partial y^2$). Symmetry arguments indicate that, if the body is symmetric about a horizontal plane, the problem may be restricted to a semi-finite domain formed by considering the symmetry plane as a rigid surface. Boundary conditions

on η are that η be Fourier transformable in the horizontal directions, and that η (x,y,0) = h(x,y), where h(x,y) is the half-body shape (including the rigid surface on which the half-body rests). The upper z-boundary condition is provided by a radiation condition of energy propagation away from the disturbance, as required by our notion of causality (see, for example, Atkinson, 1949).

To solve equation (6), we introduce the two dimensional Fourier transform

$$\hat{(\ )} \equiv \iint_{-\infty}^{\infty} (\ )\, e^{-i(\chi x + \lambda y)}\, dx\, dy \tag{7}$$

so that equation (6) is transformed to

$$\hat{\eta}'' + (\chi^2 + \lambda^2)\left(\frac{N^2}{\chi^2 U^2} - 1\right)\hat{\eta} = 0 \tag{8}$$

where primes indicate derivatives with respect to z. The solution which satisfies the boundary condition at z = 0 is then

$$\hat{\eta}(\chi, \lambda, z) = \hat{h}(\chi, \lambda)\, e^{i\mu z} \tag{9}$$

where $\mu^2 = (\chi^2 + \lambda^2)\left(\frac{N^2}{\chi^2 U^2} - 1\right)$. The vertical wave number is rendered determinate by the radiation condition. For $\chi^2 U^2 > N^2$, we must have $\hat{\eta} \to 0$ as $z \to \infty$, while for $\chi^2 U^2 < N^2$, the vertical component of group velocity $\partial\sigma/\partial\mu$, where $\sigma^2 = \chi^2 U^2 = \frac{(\chi^2 + \lambda^2)N^2}{\chi^2 + \lambda^2 + \mu^2}$ must be positive, so that

$$\begin{aligned} \mu &= \mathrm{sgn}(\chi)\sqrt{(\chi^2+\lambda^2)\left(\frac{N^2}{\chi^2 U^2} - 1\right)}\ ; \quad \chi^2 U^2 < N^2 \\ &= i\sqrt{(\chi^2+\lambda^2)\left(\frac{-N^2}{\chi^2 U^2} + 1\right)}\ ; \quad \chi^2 U^2 > N^2 \end{aligned} \tag{10}$$

Inverting the transform, we than have

$$\eta(x, y, z) = \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} \hat{h}(\chi, \lambda)\, e^{i(\chi x + \lambda y + \mu z)}\, d\chi\, d\lambda \tag{11}$$

To the linear approximation, the horizontal force on the body associated with the wave field (the wave drag) is given by

$$\underset{\sim}{D} = \iint_{-\infty}^{\infty} p(x,y,0)\, \nabla h(x,y)\, dx\, dy \tag{12}$$

and thus the rate of working against the drag or the rate of internal wave energy production is given by

$$\epsilon = \underset{\sim}{U} \cdot \underset{\sim}{D} = U \iint_{-\infty}^{\infty} p(x,y,0) \frac{\partial h}{\partial x} \, dx \, dy \tag{13}$$

By employing the equations of motion (3) - (5) and Parseval's relation (see, for example, Bremmerman, 1965, sec. 14.5) the expression (13) may be transformed to

$$\epsilon = \frac{\rho_0 U^3}{4\pi^2} \int_{-\infty}^{\infty} \int_{-N/U}^{N/U} \hat{h}^2 \frac{\varkappa^2}{(\varkappa^2 + \lambda^2)^{1/2}} \left(\frac{N^2}{U^2} - \varkappa^2 \right)^{1/2} d\varkappa \, d\lambda \tag{14}$$

The expression (14) applies only to the upper half of the body. Restricting attention to symmetric bodies, we then have that

$$\epsilon = \frac{2}{\pi^2} \rho_0 U^3 \int_0^{\infty} \int_0^{N/U} \hat{h}^2 \frac{\varkappa^2}{(\varkappa^2 + \lambda^2)^{1/2}} \left(\frac{N^2}{U^2} - \varkappa^2 \right)^{1/2} d\varkappa \, d\lambda \tag{15}$$

for the whole body.

If we consider an ellipsoidal body such that

$$\frac{x^2}{L_x^2} + \frac{y^2}{L_y^2} + \frac{z^2}{H^2} = 1 \tag{16}$$

then we have that

$$\hat{h}(\varkappa, \lambda) = \frac{4H}{L_y} \int_0^{L_x} \int_0^{L_y \sqrt{1 - x^2/L_x^2}} \left\{ L_y^2 - y^2 - x^2 L_y^2 / L_x^2 \right\}^{1/2}$$

$$\cos \varkappa x \cos \lambda y \, dy \, dx \tag{17}$$

which may be evaluated (Erdelyi et al, 1954, pp. 11 & 57) as

$$\hat{h}(\varkappa, \lambda) = \sqrt{2\pi^3} \frac{HL^{1/2}}{R} (\varkappa^2 + \lambda^2/R^2)^{-3/4} J_{3/2} \left[L (\varkappa^2 + \lambda^2/R^2)^{1/2} \right] \tag{18}$$

where we have set $R = L_x/L_y$, the aspect ratio, and $L = L_x$. J_ν is the Bessel function of the first kind of order ν. The transform (18) may be simplified somewhat by noting that the Bessel functions of half

integral order may be expressed in terms of trigonometric functions, so that

$$\hat{h}^2 = 4\pi^2 \frac{H^2}{R^2}\left(\varkappa^2 + \frac{\lambda^2}{R^2}\right)^{-2} \left\{ \frac{\sin\left[L(\varkappa^2+\lambda^2/R^2)^{1/2}\right]}{L(\varkappa^2+\lambda^2/R^2)^{1/2}} - \cos\left[L(\varkappa^2+\lambda^2/R^2)^{1/2}\right] \right\}^2 \tag{19}$$

which may be inserted into the energy production integral (15), yielding

$$\epsilon = 8\rho_0 U^3 \frac{H^2}{R^2} \int_0^{\infty}\int_0^{N/U} \left\{ \frac{\varkappa^2 (N^2/U^2 - \varkappa^2)^{1/2}}{(\varkappa^2+\lambda^2)^{1/2}(\varkappa^2+\lambda^2/R^2)^2} \right\}$$

$$\left\{ \frac{\sin\left[L(\varkappa^2+\lambda^2/R^2)^{1/2}\right]}{L(\varkappa^2+\lambda^2/R^2)^{1/2}} - \cos\left[L(\varkappa^2+\lambda^2/R^2)^{1/2}\right] \right\}^2 d\varkappa\, d\lambda \tag{20}$$

The expression (20) is nondimensionalized by defining

$$k = \varkappa U/N$$

$$l = \lambda U/RN$$

$$\tau_u/\tau_b = NL/U$$

and introducing the normalization factor (Miles, 1969a)

$$E = \epsilon \Big/ \frac{\pi}{2}\rho_0 U^2 N H A_0$$

where A_0 is the frontal area, equal to $\pi HL/R$ for the ellipsoid. In normalized form, then, we have

$$E = \frac{16}{\pi^2}\left(\frac{\tau_u}{\tau_b} R\right)^{-1} \int_0^{\infty}\int_0^{1} \frac{k^2(1-k^2)^{1/2}}{(k^2/R^2+l^2)^{1/2}(k^2+l^2)^2}$$

$$\left\{ \frac{\sin\left[\frac{\tau_u}{\tau_b}(k^2+l^2)^{1/2}\right]}{\frac{\tau_u}{\tau_b}(k^2+l^2)^{1/2}} - \cos\left[\frac{\tau_u}{\tau_b}(k^2+l^2)^{1/2}\right] \right\}^2 dk\, dl \tag{21}$$

for the rate of internal wave energy production by the steady horizontal motion (time scale τ_u = L/U) of an ellipsoid with length-width aspect ratio R in a uniformly stratified fluid (time scale τ_b = 1/N).

The energy production integral (21) may be evaluated numerically for various values of R and τ_u/τ_b. Fig. 2 is a comparison between the energy production rate for an ellipsoid with R = 1 with that for other three dimensional bodies for which computations have appeared in the literature (Blumen, 1965, Mackinnon et al, 1969). A striking qualitative difference between the curves is apparent for $\tau_u/\tau_b \lessapprox 1/2$, the production rate for the ellipsoid being markedly larger than that for

the other bodies within this range, with the curvature of the E vs τ_u/τ_b curves of opposite sign. The structure for ellipsoids of various R is illustrated for this range of τ_u/τ_b in Fig. 3. This qualitative departure is tentatively attributed to the breakdown of the linearized boundary condition around the lateral edges of the ellipsoid. The bluff sides of the ellipsoid require infinite vertical velocities at the edge in linear theory, while the other bodies, although of infinite extent, are smooth in the appropriate lateral directions, resulting in finite vertical velocities which tend uniformly to zero as the height parameter tends to zero*. A qualitative difference is also apparent in Fourier space, where, because of the slope discontinuity, the lateral transform of the ellipsoid falls off as λ^{-2}, while that for the other (infinitely smooth) bodies falls off exponentially. This slow decay associated with the slope discontinuity may in fact dominate the behavior for small τ_u/τ_b. However, as will become apparent later, the behavior of the energy production rate curve in this parameter range is not of crucial importance, while that for larger τ_u/τ_b, which is illustrated in Fig. 4 for various values of R, appears to be reliable, provided the height scale H of the body is sufficiently small so that $F^{-1} \lesssim 1/2$. For prolate spheroids (ellipsoids of revolution), this requirement reduces to $\tau_u/\tau_b \lesssim \frac{1}{2} R$. Although there is no a priori justification for the extrapolation of two dimensional results to the three dimensional case, it should be noted that, in the two dimensional problem, the effect of at least mild nonlinearity, such that upstream influence and separation (see, for example, Miles 1971) are not brought into play, is to increase internal wave energy production rates above those predicted by linear theory (Miles, 1969). For very strong nonlinearity ($\tau_u/\tau_b \gg R$) we expect a reduction in the energy supply to the wave field, since in this limit the flow is confined predominantly to horizontal planes (Drazin, 1961, Grimshaw, 1969).

Wake collapse effect

In a real fluid, as the flow proceeds around the body, a boundary layer of retarded flow develops which, in the cases of physical interest, rapidly becomes unstable and degenerates into turbulence. Within this turbulent boundary layer and its extension as a turbulent wake, turbulent diffusion acts to reduce the ambient density gradient, creating a gravitationally unstable situation, which is rectified by the release of the stored potential energy into kinetic energy of collapse as the wake is flattened vertically. Disturbance energy associated with this collapse process radiates into the surrounding fluid in the form of an internal gravity wave field. Although some theoretical progress based primarily on traditional dimensional and similarity arguments, has been made on the problem of turbulent wakes in density stratified media (Onufriyev, 1970, Ko, 1971; see also Monin and Yaglom, 1971, ch. 4),

*For the bodies considered by Mackinnon et al (1969), the height parameter is in fact the half-width of the body. See appendix for a discussion of an approximate approach which removes the difficulty associated with the bluff sides of the ellipsoid.

a complete description of the process of wake generation, growth, and collapse, including the interaction of the wake with the surrounding fluid, is not available. We can, however, obtain bounds on the rate at which energy is supplied to the wave field by invoking rather basic similarity and dimensional arguments and relying rather heavily on available empirical data.

Wake collapse is a continuous process, active whenever buoyancy induced potential energy exists within the wake region. However, wake potential energy is created at the expense of turbulence kinetic energy, which is in turn fed by mean flow velocity gradients within the wake region, so that sensible creation of wake potential energy is confined to a finite distance behind the body. This aspect of the phenomenon is reflected in the observation that a dyed wake expands to a maximum vertical size and then flattens out. After the passage of the point of maximum vertical expansion, we can expect that sensible potential energy production is sharply curtailed, so that the amount of potential energy created during the expansion stage should be proportional to the rate of internal wave energy production.

We may estimate the amount of available potential energy by considering the wake cross section at the point of maximum vertical expansion. As a model, we may consider an ellipsoidal figure, with horizontal major axis, within which there is a uniform stratification with Brunt-Vaisala frequency N_i, the surrounding fluid being characterized by the Brunt-Vaisala frequency N, so that we may define a mixing factor

$$f_M = \frac{N^2 - N_i^2}{N^2} \tag{22}$$

In order to account for potential energy which has been lost during the expansion stage, we replace the ellipse by a circle with equal area. The potential energy created per unit wake length may then be estimated as the work required to create this partially mixed circular area of fluid by distorting the ambient density field. Referring to Fig. 5, the displacement δ of a fluid parcel from its equilibrium position is given by

$$\delta = f_M z \tag{23}$$

The buoyancy force acting on a fluid element whose equilibrium position is z_E is given by

$$F = \rho_o N^2 (z_E - z) \tag{24}$$

so that the potential energy density within the wake cross section is given by

$$\begin{aligned} \Delta &= \rho_o N^2 \int_{z_E}^{z_E+\delta} (z - z_E)\, dz \\ &= \frac{1}{2} \rho_o N^2 \delta^2 \end{aligned} \tag{25}$$

and the total potential energy per unit wake length is

$$P = \frac{\rho_o}{2} N^2 \int_A \delta^2 dA \tag{26}$$

where the integral is over the cross sectional area. For the circular partially mixed region, we thus have

$$P = 2\rho_o N^2 f_M^2 \int_0^r \int_0^{\sqrt{r^2-x^2}} z^2 \, dz \, dx$$

$$= \rho_o \frac{\pi}{8} N^2 f_M^2 r^4 \tag{27}$$

where r is the radius of the circle. The energy per unit wake length supplied to the internal wave field should then be proportional to this. Choosing the constant of proportionality to be unity, the rate of internal wave energy production by the wake collapse mechanism is approximated by

$$\epsilon_W = \frac{\pi}{8} \rho_o N^2 U f_M^2 r^4 \tag{28}$$

Normalizing as before, we then have that

$$E_W = \frac{1}{4\pi} \frac{N}{U} f_M^2 \frac{r^4}{H^3} \tag{29}$$

where H is the body radius. If H_o is a characteristic length scale for the wake, such as the body radius ($H_o = H$ in this case) or a propeller radius in the case of self-propulsion, then we have that

$$\left(\frac{H}{H_o}\right)^4 E_W = \frac{1}{4\pi} \frac{T_u}{T_b} R^{-1} f_E^2 f_M^2 \tag{30}$$

where $R = L/H$ and $f_E = r^2/H_o^2$, the ratio of the wake cross sectional area at maximum vertical expansion to the characteristic wake area. In order to proceed further, we must invoke basic similarity arguments. If we assume that the potential energy, which derives from the wake kinetic energy, scales by the external parameters U and H_o, then we have that

$$\frac{\pi}{8} \rho_o N^2 U f_M^2 r^4 \propto \rho_o H_o^2 U^3 \tag{31}$$

so that the scaling requires

$$F_W^{-1} f_E f_M = const. \tag{32}$$

where F_W^{-1} is the inverse internal Froude number based on the wake

characteristic scale H_O. Thus, we arrive at the relationship

$$E_W \propto \left(\frac{H_o}{H}\right)^2 R \left(\tau_u/\tau_b\right)^{-1} \tag{33}$$

for the normalized rate of internal wave energy production by the wake collapse mechanism.

In order to evaluate the constant of proportionality, recourse must be made to empirical data. Fig. 6 is a plot of f_E as a function of F_W^{-1}, based on available data (Schooley and Stewart, 1963, Stockhausen et al, 1966; unpublished data provided by K. G. Williams of NRL) from experimental studies with self-propelled bodies*, the self-propulsion mode being the most physically relevant. Although there is no theoretical justification for such a choice, the data is consistent with a relationship of the form

$$f_E \doteq 1.5 \, F_W^{1/2} \tag{34}$$

Examination of Fig. 7, which illustrates normalized vertical density profiles near the vertical centerline of a wake at maximum vertical expansion based on the data in the report by Stockhausen et al (1966), indicates that an equivalent uniformly mixed wake would have $N_1^2 \approx 1/2$, for $F_W^{-1} \approx 0.04$. Thus, based on the relationships (32) and (34) and this single piece of data, we obtain the approximate relationship

$$f_M \doteq 0.1 \, F_W^{1/2} \tag{35}$$

It is important to emphasize that there is no theoretical basis for the expressions (34) and (35), although they are consistent with the available data and the relationship (32) derived from similarity arguments. In deriving (32), what in fact we have done is to replace the generating body with a virtual wake source, so that when results which are inconsistent with the reality of the body are encountered, the validity of the model comes into question. The basic constraints which must be satisfied if the model is to be at least internally consistent are that $f_M \ll 1$ and $f_E \gg 1$, which introduces the bounds

$$0.01 \lesssim F_W^{-1} \lesssim 2.25 \tag{36}$$

Within these bounds, we then have an approximate expression

$$E_W \doteq 0.002 \left(\frac{H_o}{H}\right)^2 \frac{R}{\tau_u/\tau_b} \tag{37}$$

*The experiments by Williams involved simulated self-propulsion, in which a model with attached propeller was towed thru a tank, the propeller being driven at a rate such that the pitch of the propeller times the rotation rate was equal to the forward speed of the model.

for the rate of internal wave energy production by the wake collapse process.

The normalized rate of energy supply to the internal wave field as determined above is illustrated in Fig. 8. Included in Fig. 8 is an indication of the relative location of the data points which went into determining the expressions (34) and (35). The important feature of the assumed functional form for the relationship (33) is the inverse proportionality of E_W and τ_i / τ_b , which is at least consistent with the idea that a faster body will create a more energetic wake, and the expression (37) should provide a reasonable estimate of the rate of internal wave generation, at least over the parameter ranges covered by the model studies.

The displacement thickness effect

The presence of the boundary layer of retarded fluid adjacent to the body causes an outward displacement of streamlines in the surrounding fluid which is perpetuated by the turbulent wake and results in an effective increase in the size of the body over that presented to the system in the absence of viscous effects. The dynamics of such near field flow processes will be largely decoupled from the ambient stratification provided the characteristic time scale τ_* of the process is small compared to that associated with the environmental density gradient $\tau_b = N^{-1}$. If we assume that $\tau_* \ll \tau_b$, and if the body is essentially axisymmetric, then the mean flow in the near field may be described by the Stokes stream function ψ , where

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial r} \tag{38a}$$

$$v = \frac{1}{r}\frac{\partial \psi}{\partial x} \tag{38b}$$

where u and v are the axial (x direction) and radial (r direction, where $r^2 = y^2 + z^2$) velocity components. Cylindrically shaped surfaces of constant ψ are the Stokes stream surfaces, which are the axisymmetric analog of streamlines in two-dimensional flows.

A number of parameters have been developed to describe the properties of boundary layers and wakes. For the purposes of this report, two integral parameters are of interest, the displacement thickness δ_1, and the momentum loss thickness δ_2 , where we define*

* These definitions differ somewhat from the more traditional definitions which are carry-overs from two dimensional theory. The present definitions are used to facilitate interpretation in the sequel. A thorough discussion of the concept of displacement thickness is given by Lighthill (1958).

$$\frac{1}{2} U \delta_1^2 = \int_{r_b}^{\infty} r(U-u)\,dr \tag{39a}$$

$$\frac{1}{2} U^2 \delta_2^2 = \int_{r_b}^{\infty} r u (U-u)\,dr \tag{39b}$$

where r_b is the local body radius and u is the axial flow speed in the boundary layer. Since we are primarily interested in the effects associated with the turbulent boundary layer and its extension as the turbulent wake, it is convenient and conceptually advantageous to replace the body with a momentum source along r = 0 which will produce an equivalent boundary. In this case, $r_b = 0$ and, to the boundary layer approximation*, the displacement thickness δ_1 may be interpreted as an effective outward displacement of the Stokes stream surface $\psi = 0$, insofar as the flow outside the boundary layer is concerned. To illustrate this interpretation, consider a flow such that $u = U$ for $r > \delta$. For $r > \delta$, the stream function ψ is given by

$$\psi = -\int_0^{\delta} r' u\,dr' - \int_{\delta}^{r} r' U\,dr'$$

$$= \frac{1}{2} U \delta_1^2 - \frac{1}{2} U r^2$$

If, however, the stream surface $\psi = 0$ were situated at $r = \delta_1$, instead of r = 0, and the flow were uniform ($u = U$), we would have

$$\psi = -\int_{\delta_1}^{r} r' U\,dr'$$

$$= \frac{1}{2} U \delta_1^2 - \frac{1}{2} U r^2$$

Thus, the displacement thickness is interpreted as the radius of an equivalent body in potential flow. This is the "displacement body". The momentum loss thickness δ_2 is a measure of the frictional drag on the upstream portion of the body.

A number of techniques for computing turbulent boundary layers in general are available (see, for example, Kline et al, 1969) some of which possess fairly straightforward generalizations to boundary layers on axisymmetric bodies (see, for example, Tsakonas and Jacobs, 1960, Yamaoka, 1963, Nelson, 1964, 1966, Tetervin, 1969). If the body is self-propelled, the computation is rendered considerably more complicated by the pressure gradients near the tail of the body induced by

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* See sections IIa and XIXa of Schlichting's (1968) book for a discussion of the boundary layer approximation.

the propeller. In this case, iterative procedures (see, for example, Amtsberg, 1960, Hucho, 1969; see also Beveridge, 1969) may be used to compute boundary layers which compare favorably with experiment over all but the very tail end of the body, in the immediate vicinity of the propeller.

Behind the body, the turbulent boundary layer is extended as a turbulent wake. In the absence of self-propulsion, the wake is characterized to the boundary layer approximation by a constant momentum loss thickness. The condition of self-propulsion, on the other hand, drastically alters the integral properties of the boundary layer. Again to the boundary layer approximation, the condition of self propulsion requires that $\delta_2 = 0$, that is, that there is no net drag on the body-propeller system. A complete description of the interaction between the boundary layer and the propeller is not possible with the present state of the art of hydrodynamic theory. However, since we are primarily interested in the displacement thickness of the wake, which is an integral property and hence should be somewhat insensitive to the details of the flow structure, we may estimate the change in δ_1 on passing thru the region of strong propeller influence in a fairly simple manner, using the condition of self propulsion.

A slight distance upstream of the propeller, the displacement and momentum thicknesses are δ_1 and δ_2 , while downstream, the displacement thickness is δ_{1F} , with $\delta_{2F} = 0$. Representing the downstream velocity as $u + u_p$, where u is the upstream velocity profile and u_p is the modification thereof induced by the propeller, we have

$$\frac{1}{2} U \delta_1^2 = \int_0^\infty (U-u)\, r\, dr$$

$$\frac{1}{2} U^2 \delta_2^2 = \int_0^\infty u(U-u)\, r\, dr$$

$$\frac{1}{2} U \delta_{1F}^2 = \int_0^\infty (U-u-u_p)\, r\, dr$$

$$\frac{1}{2} U^2 \delta_{2F}^2 = \int_0^\infty (u+u_p)(U-u-u_p)\, r\, dr$$

If we represent u_p as a uniform velocity over the upstream boundary layer thickness δ , where U - u = 0 for r $> \delta$, then we have that

$$\frac{1}{2} U \delta_{1F}^2 = \frac{1}{2} U \delta_1^2 - \frac{1}{2} u_p \delta^2$$

$$\frac{1}{2} U^2 \delta_{2F}^2 = \frac{1}{2} U^2 \delta_2^2 + \frac{1}{2} u_p U \delta_1^2 - \frac{1}{2} u_p^2 \delta^2 + \frac{1}{2} u_p U \delta_1^2 - \frac{1}{2} u_p U \delta^2$$

The condition of self propulsion ($\delta_{2F} = 0$) then requires that

$$u_p = \frac{1}{2} U \left\{ \left(\frac{\delta_1}{\delta}\right)^2 - 1 + \sqrt{\left[\left(\frac{\delta_1}{\delta}\right)^2 - 1\right]^2 + 4\left(\frac{\delta_2}{\delta}\right)^2} \right\} \tag{40}$$

In practice, δ may be of order 30% to 50% of the maximum body radius H, δ_1 may be 15% to 25% of H, and the shape parameter $H_{12} = \delta_1/\delta_2$ will generally be greater than 1.3 and may be greater than 2 (Burstein, 1965, Reed et al, 1966, Hucho, 1969), so that u_p may be from 7% to 10% of U, and δ_{1F} may be from 10% to 20% of the maximum body radius.*

As noted previously, the dynamics of turbulent wake development in a stratified medium is not fully understood. However, for the momentumless wake in a homogeneous fluid, an extensive experimental investigation exists, due to Naudascher (1965). At sufficiently large distances downstream of the generating element, Naudascher found an essentially self-preserving wake form. Taking advantage of the self-preserving quality of the wake, we may use Naudascher's data to examine the change in displacement thickness with distance downstream. Fig. 9 illustrates the decay of the normalized displacement thickness $A\,\delta_1/H_o$ as a function of nondimensional distance downstream x/H_o, where A is a constant which depends on the exact shape of the self-preserving form of the wake. Extrapolation of Naudascher's data indicates that the displacement thickness is reduced essentially to zero at a downstream distance $x/H_o = 150$. This indicates that for a self-propelled body with length/diameter aspect ratio R = 7 1/2 and $H_o/H = 1/2$ in a homogeneous fluid, the displacement body associated with the turbulent wake would be approximately 5 times as long as the generating body, although as indicated previously, the maximum radius of the displacement body would be only 10% or 20% of the maximum radius of the generating body.

In order to assess the relative importance of the displacement thickness effect, we must consider its interaction with the hull and wake collapse effects. In obtaining estimates of the size of the effective displacement body associated with the turbulent boundary layer and wake, we have been forced to rely on data and arguments which are strictly applicable only to homogeneous fluids. If the time scale $\tau_* = L_*/U$, where L_* is the half-length of the effective displacement body, becomes comparable to τ_δ, we may expect some distortion of the displacement body due to buoyancy effects. This distortion should be reflected in a distortion of the turbulent wake structure which is similar to but distinct from that associated with the wake

*For model studies in which the propeller radius is nearly equal to the body radius, δ_{1F} may be a considerably larger fraction of the body radius.

collapse mechanism, although in general we would expect the effect of this interaction to be of minor importance. Potentially more important is the interaction with the hull effect. An examination of Fig. 4 indicates that the presence of an effective extension of the generating body will enhance the supply of energy to the hull effect internal wave field. The exact magnitude of the relative increase in hull wave making propensity would be difficult to assess with the relatively crude models discussed in this report. For the parameter range considered above (maximum displacement thickness of approximately 1/2 H_0, $H_0/H \cong 1/4$) the increase should be slight, although for model studies for which H_0/H is generally of order unity the effect may be sensible, and significant increases may be found for non-self propelled models for which, due to the absence of the propeller contraction, the maximum displacement thickness is greater, and for which the characteristic length of the displacement body is substantially increased (see, for example, Tennekes and Lumley, 1972, sec. 4.2 and 4.3).

Conclusions

We have systematically investigated the basic physical processes which are associated with the passage of a body thru a stratified fluid and which result in disturbance motions which are steady in a reference frame moving with the body. Three dominant mechanisms are involved in the flux of energy into the internal wave field which is stationary with respect to the body. The first two mechanisms (the hull effect and the wake collapse effect) act somewhat independently, and have different parametric spheres of influence. For a given body shape and size and a given environmental stratification, the body effect dominates the wave field at relatively low speeds, while the wake collapse effect dominates at relatively high speeds. For example, if we consider a body with characterisitc half-length L = 50 m and length-breadth aspect ratio R = 10, travelling in a fluid with $N \approx 10^{-2} sec^{-1}$, then, assuming a propulsive system such that the characteristic wake radius is 1/2 the characteristic body radius, we may expect the body effect to dominate the radiated wave field for speeds less than approximately 1 m/sec, while the wake collapse effect should dominate for speeds in excess of 2 m/sec. The third effect is important insofar as it interacts with the other effects, the dominant interaction occurring with the hull effect. This mechanism, the displacement thickness effect, produces an effective extension of the body, and a consequent increase in the supply of hull effect type energy to the internal wave field. Although the net increase in radiated energy due to this effect is probably of minor importance in situations similar to that described in the above example, it may be of significant importance in model studies which do not involve self propulsion, resulting in a misleading amplification of hull-type internal wave energy in such situations.

Appendix

Problems associated with the bluff sides of an ellipsoid may be satisfactorily, albeit not rigorously, resolved by considering a new body obtained by multiplying the local height of the ellipsoid by the vertical projection of the outward normal of the ellipsoid at that point. The intuitive justification of such an approximation arises from the fact that linear theory accounts directly for only the vertical component of induced velocity. Thus, we replace the body shape

$$\frac{z^2}{H^2} = 1 - \frac{x^2}{L^2} - \frac{y^2}{H^2}$$

by the new shape

$$\frac{z^2}{H^2} = \left(1 - \frac{x^2}{L^2} - \frac{y^2}{H^2}\right)^2 \qquad \text{A1}$$

for which the induced vertical velocity vanishes around the edge.

We may proceed as before, obtaining the Fourier transform of this modified body shape

$$\hat{h}(\chi,\lambda) = 8\pi L \frac{J_2[L(\chi^2+\lambda^2/R^2)^{\frac{1}{2}}]}{L^2(\chi^2+\lambda^2/R^2)} \qquad \text{A2}$$

so that the rate of internal wave energy production is

$$\epsilon = 128\rho_0 U^3 H^4 L^2 \int_0^\infty \int_0^{N/U} \frac{\chi^2(N^2/U^2-\chi^2)^{1/2}}{(\chi^2+\lambda^2)^{1/2}} \left\{\frac{J_2[L(\chi^2+\lambda^2/R^2)^{1/2}]}{L^2(\chi^2+\lambda^2/R^2)}\right\}^2 d\chi\, d\lambda \qquad \text{A3}$$

Nondimensionalizing and normalizing as before, we then have that

$$E = \frac{256}{\pi^2}\left(\frac{\tau_u}{\tau_b}\right)^3 R^{-1} \int_0^\infty \int_0^1 \frac{k^2(1-k^2)^{1/2}}{(k^2/R^2+l^2)^{1/2}} \left\{\frac{J_2[\frac{\tau_u}{\tau_b}(k^2+l^2)^{1/2}]}{\left(\frac{\tau_u}{\tau_b}\right)^2(k^2+l^2)}\right\} dk\, dl \qquad \text{A4}$$

This expression would then replace the expression (21) for the normalized rate of internal wave energy production associated with the hull effect.

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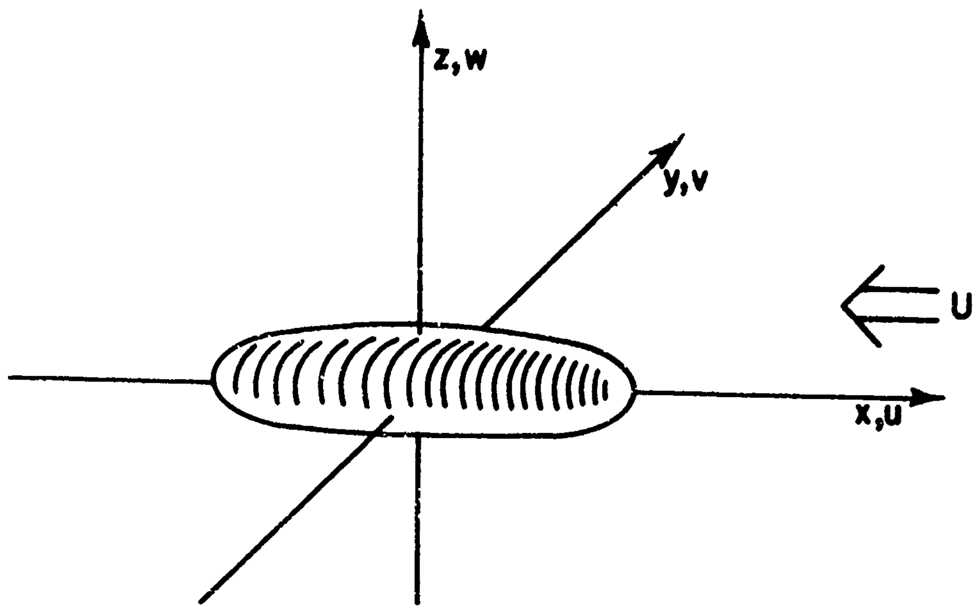

Fig. 1 - Definition sketch

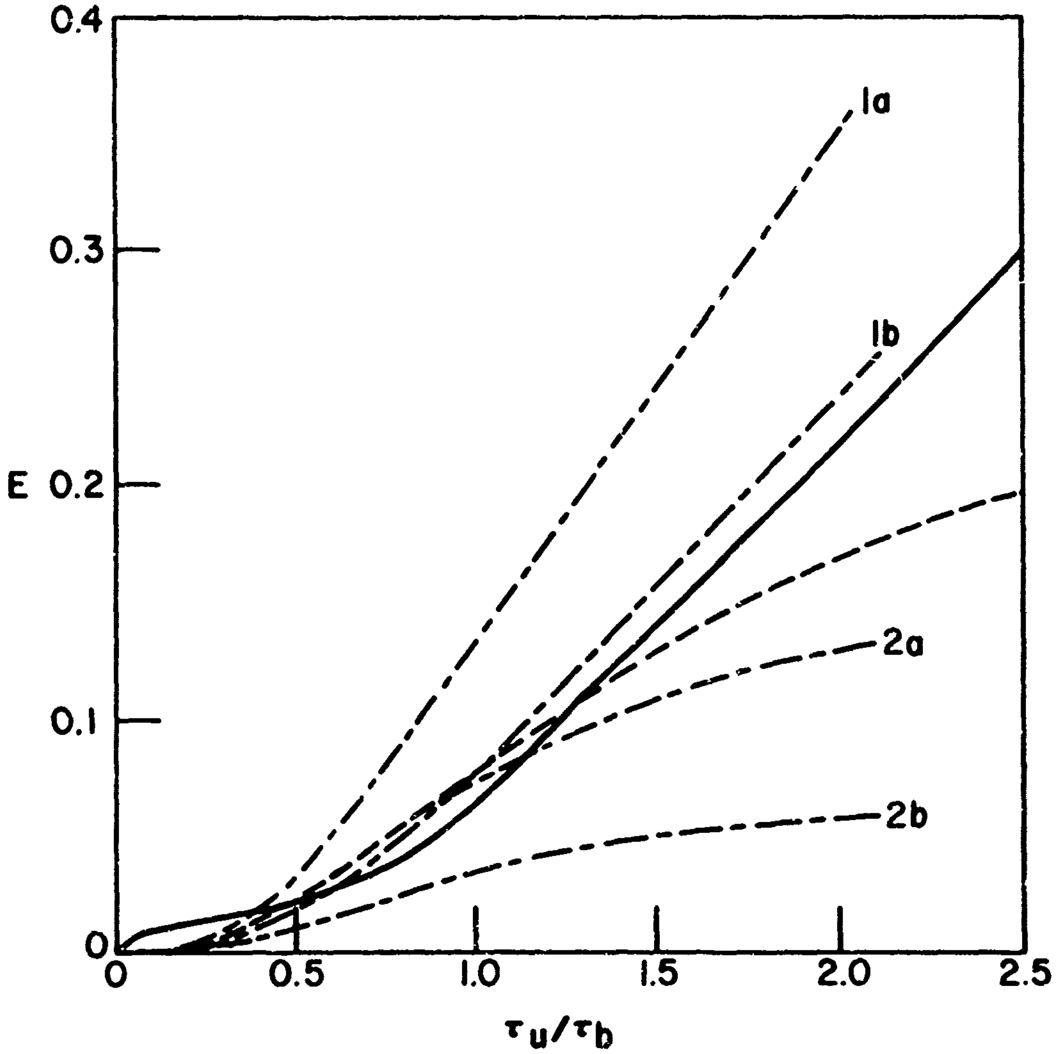

Fig. 2 - Normalized internal wave energy production rates (E) as a function of the ratio τ_u / τ_b = NL/U for various three dimensional bodies.

——— ellipsoid: $\frac{z^2}{H^2} = 1 - \frac{x^2}{L^2} - \frac{y^2}{H^2}$

- - - generalized Witch (Blumen 1965): $\frac{z^2}{H^2} = \left(\frac{x^2}{L^2} + \frac{y^2}{L^2} + 1\right)^{-3}$

—·—·— vertical discs (after Mackinnon et al 1969):

body #1. $\frac{y^2}{L^2} = \exp\left\{-2\left(\frac{x^2}{L^2} + \frac{z^2}{H^2}\right)\right\}$

body #2. $\frac{y^2}{L^2} = \left\{\left(1 + \frac{x^2}{L^2}\right)\left(1 + \frac{z^2}{H^2}\right)\right\}^{-2}$

for ratios H/L = 0.1 (a) and 10.(b).

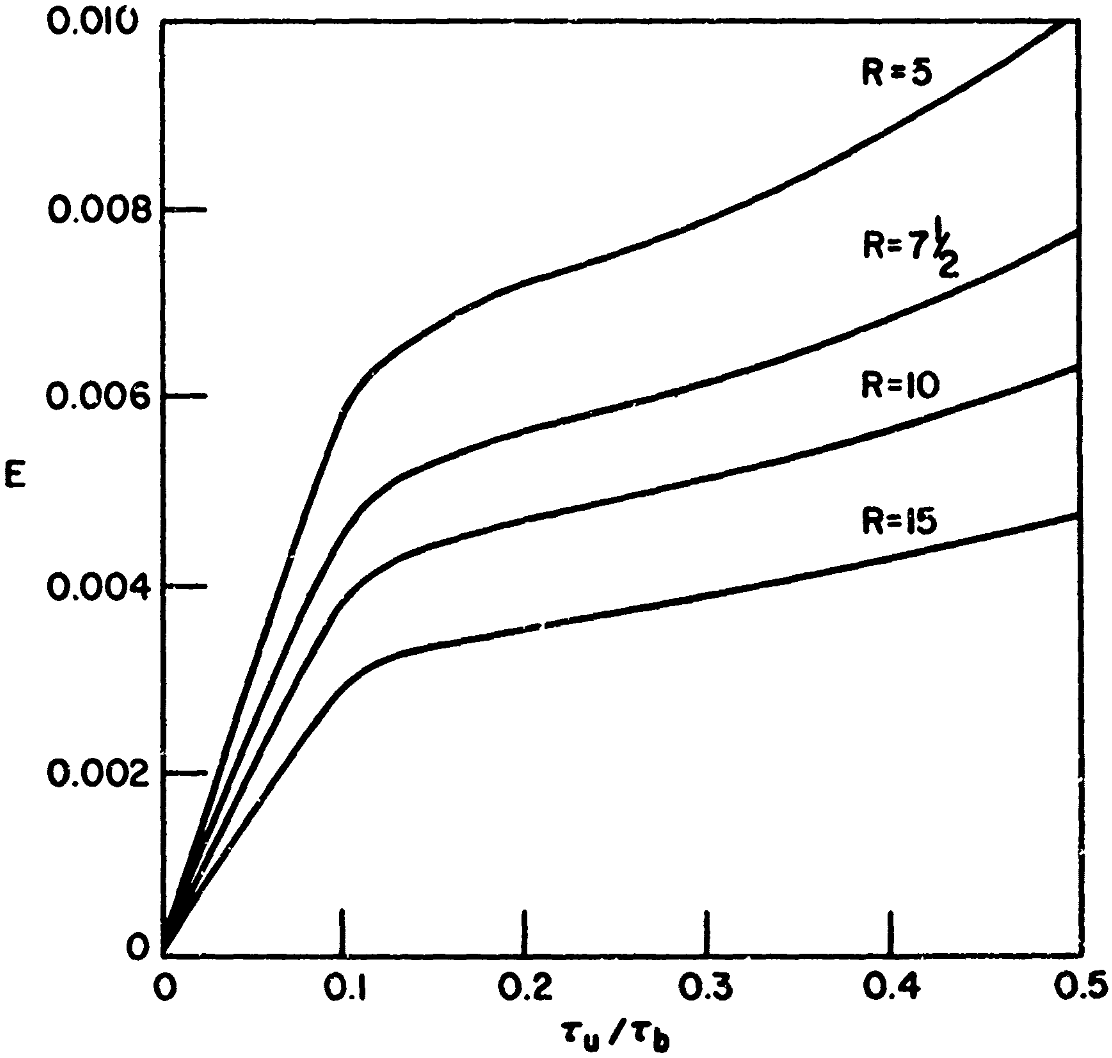

Fig. 3 - Anomalous behavior of normalized energy production rate (E) for small values of the ratio τ_u/τ_b = NL/U for ellipsoids with aspect ratio R. It is expected that production rates illustrated in this figure are generally larger than those actually realized (see text).

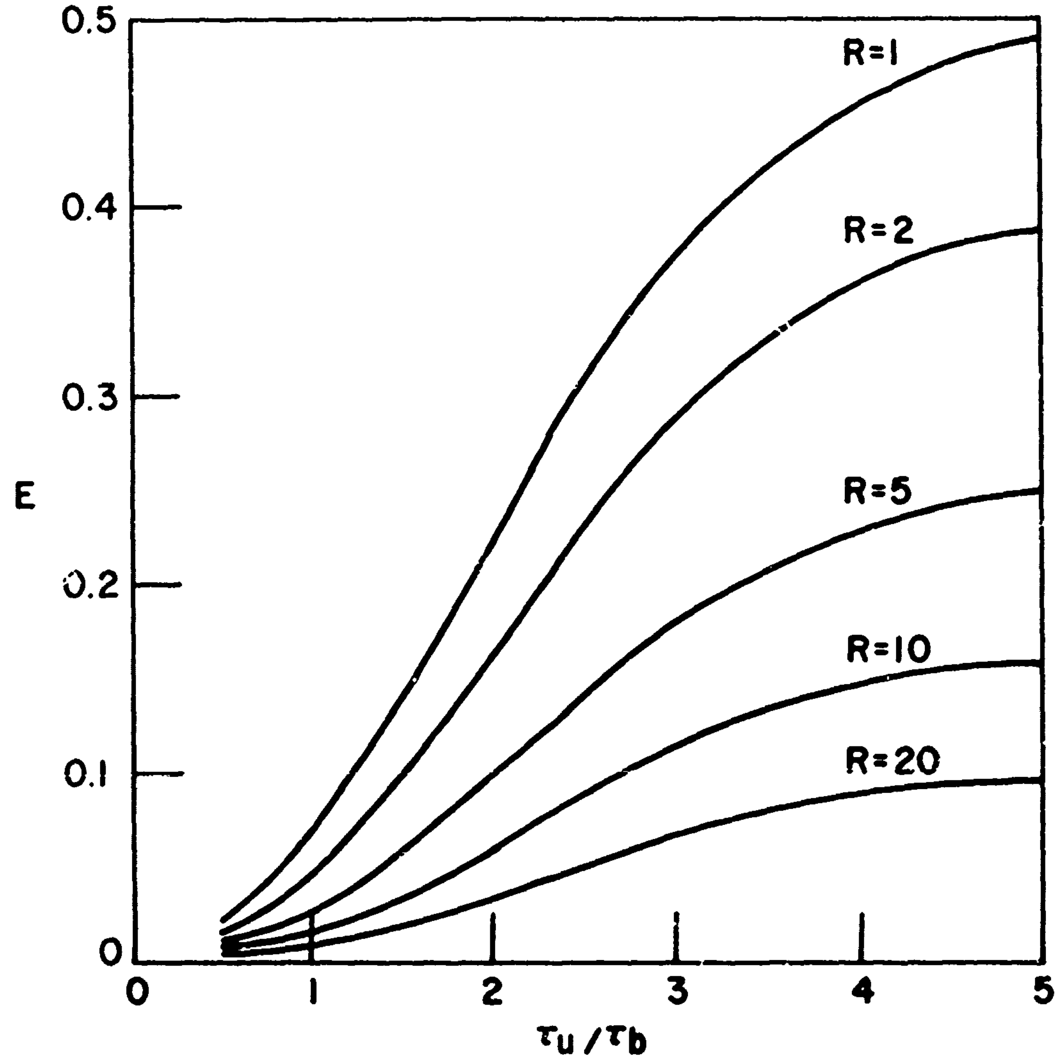

Fig. 4 - Normalized internal wave energy production rate (E) as a function of the ratio τ_u / τ_b = NL/U for ellipsoids with aspect ratio R. Production rates illustrated in this figure are expected to be reliable provided that $R^{-1}\tau_u/\tau_b < 1/2$.

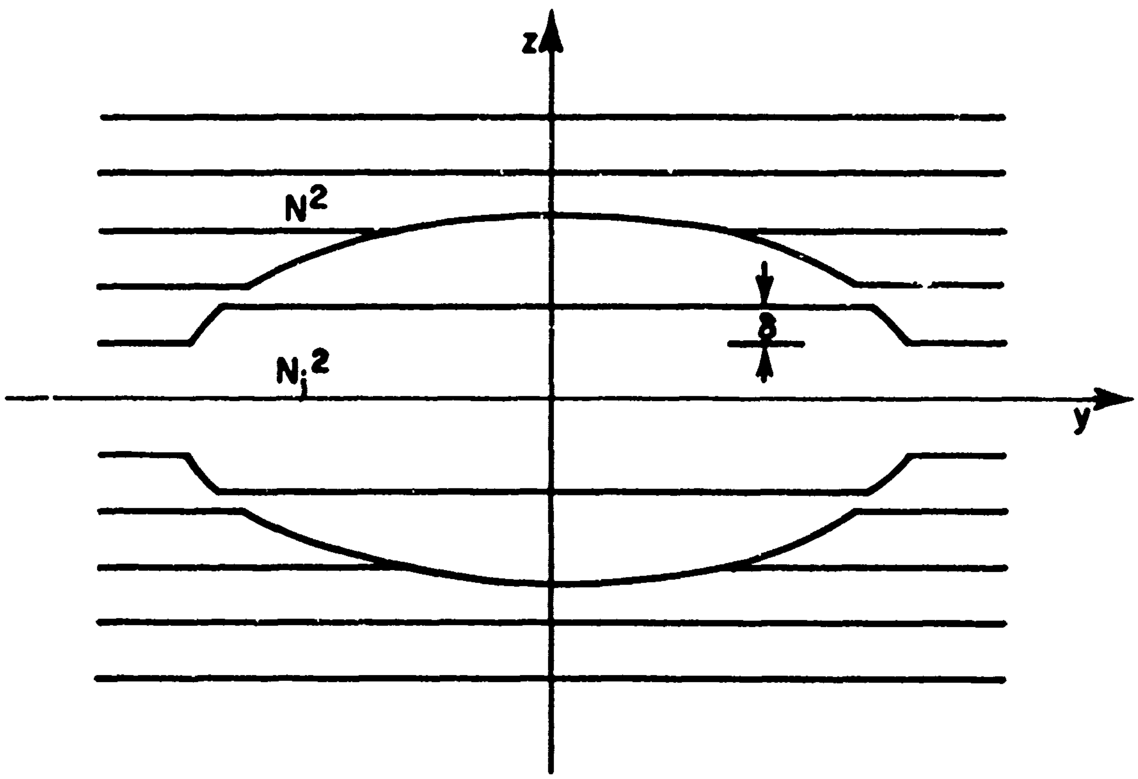

Fig. 5 - Idealized model of cross-section of partially mixed wake, as defined by isopychnals, illustrating the displacement function δ. Stratification inside mixed region has Brunt-Vaisala frequency N_i, that in environment has N.

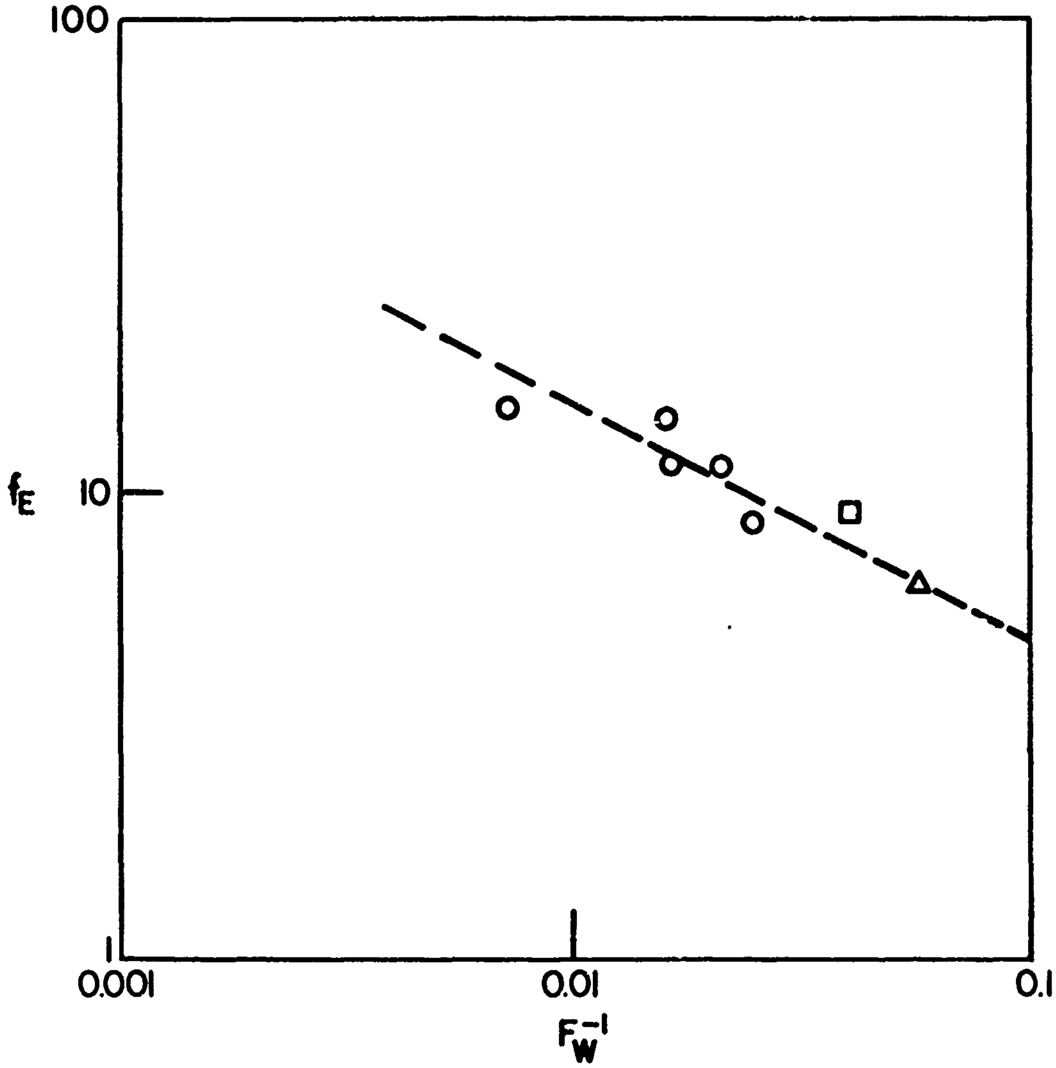

Fig. 6 - Wake expansion factor ($f_E = A_E/A_o$, where A_E is the cross-sectional area at maximum vertical expansion and A_o is the initial or characteristic area) as a function of $F_W^{-1} = NH_o/U$, where H_o is the initial radius. The dashed line has a slope of -1/2.

□ - Schooley & Stewart (1963)

Δ - Stokhausen, Clark & Kennedy (1966)

o - Williams (unpublished)

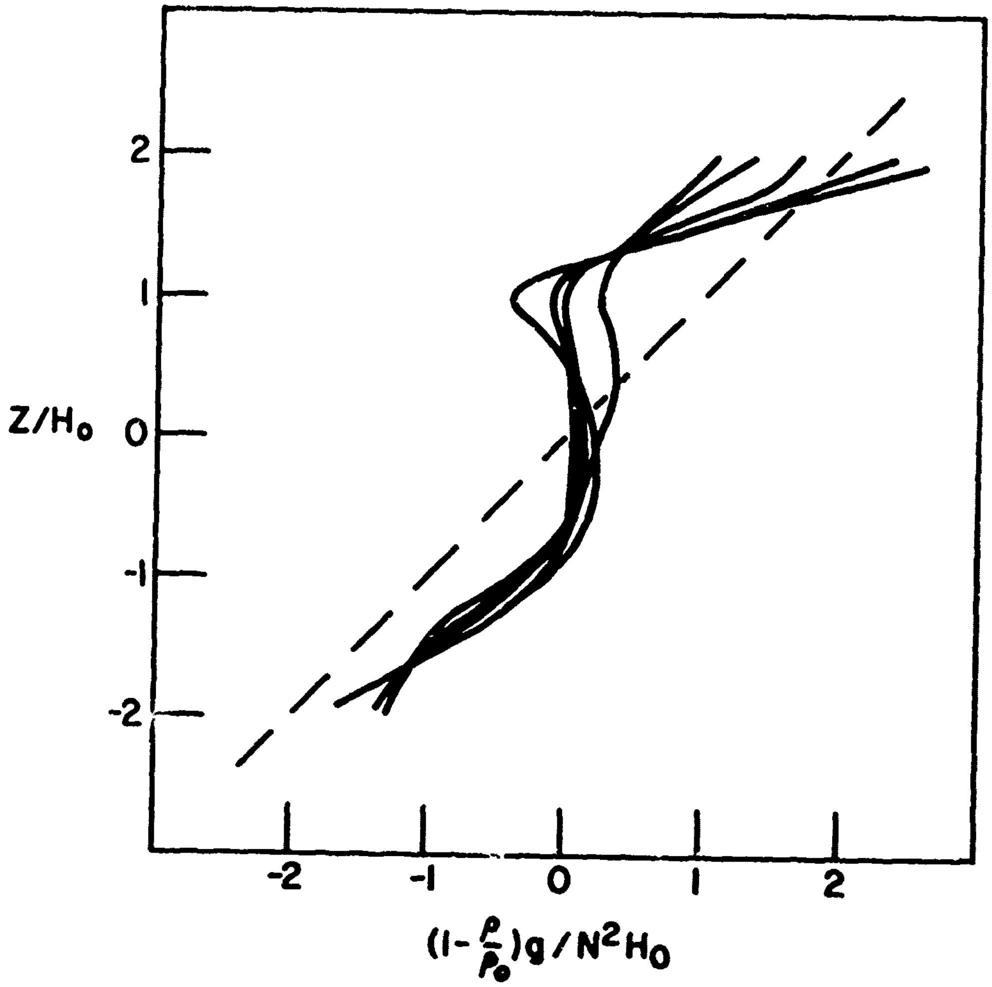

Fig. 7 - Normalized vertical density profiles near the vertical centerline of a wake at maximum vertical expansion, $Fw^{-1} = 0.04$; dashed diagonal line is undisturbed profile. (after Stockhausen, Clark & Kennedy, 1966)

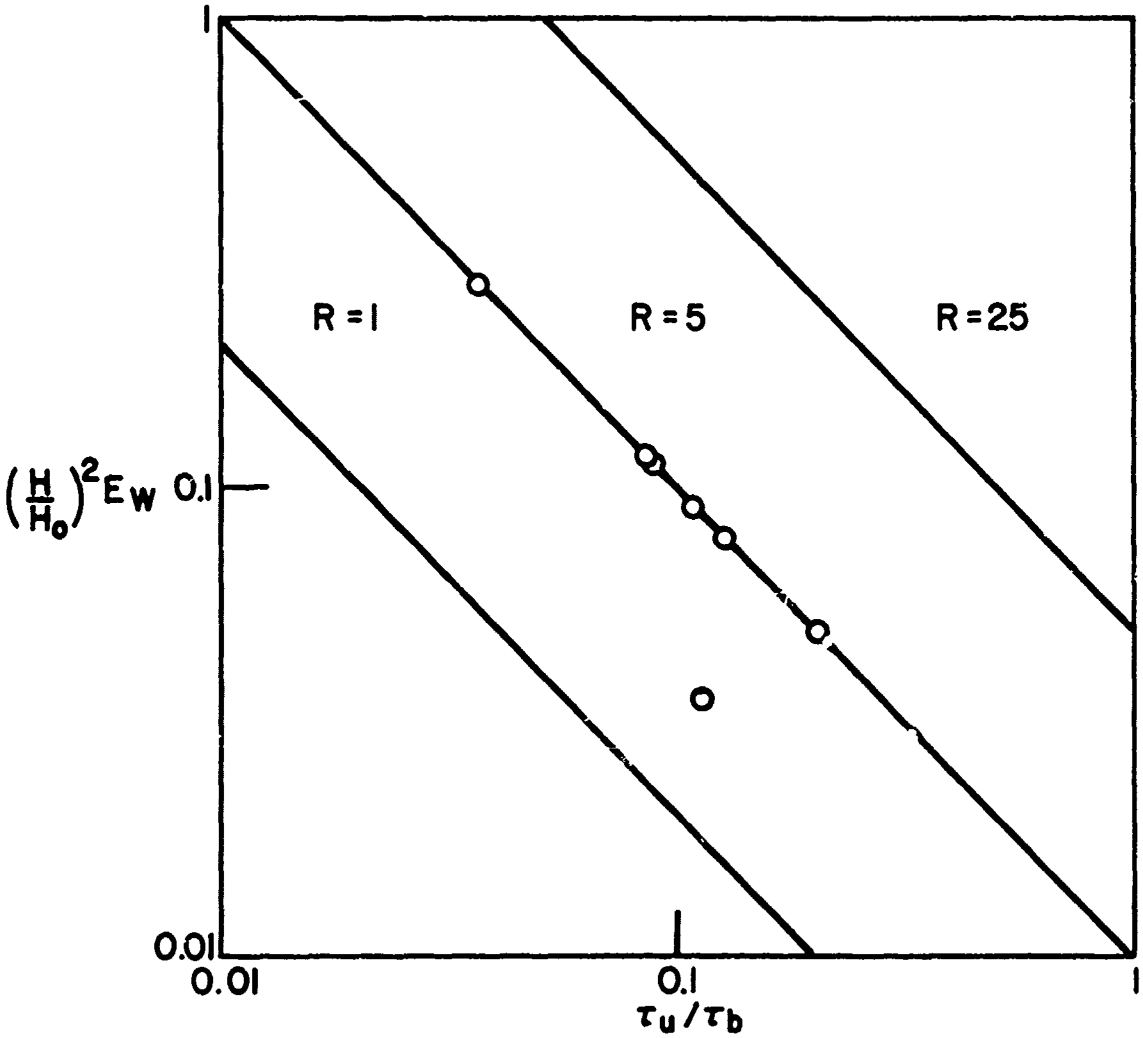

Fig. 8 - Normalized power (E_W) associated with wake potential energy density as a function of τ_u/τ_b = NL/U, for various body aspect ratios (R = L/H), where H/H_o is the ratio of body radius to characteristic initial wake radius. Normalization is the same as that used for "hull effect" internal wave energy production rate. Points indicate parametric location of data used in evaluating the constant of proportionality in expression (31).

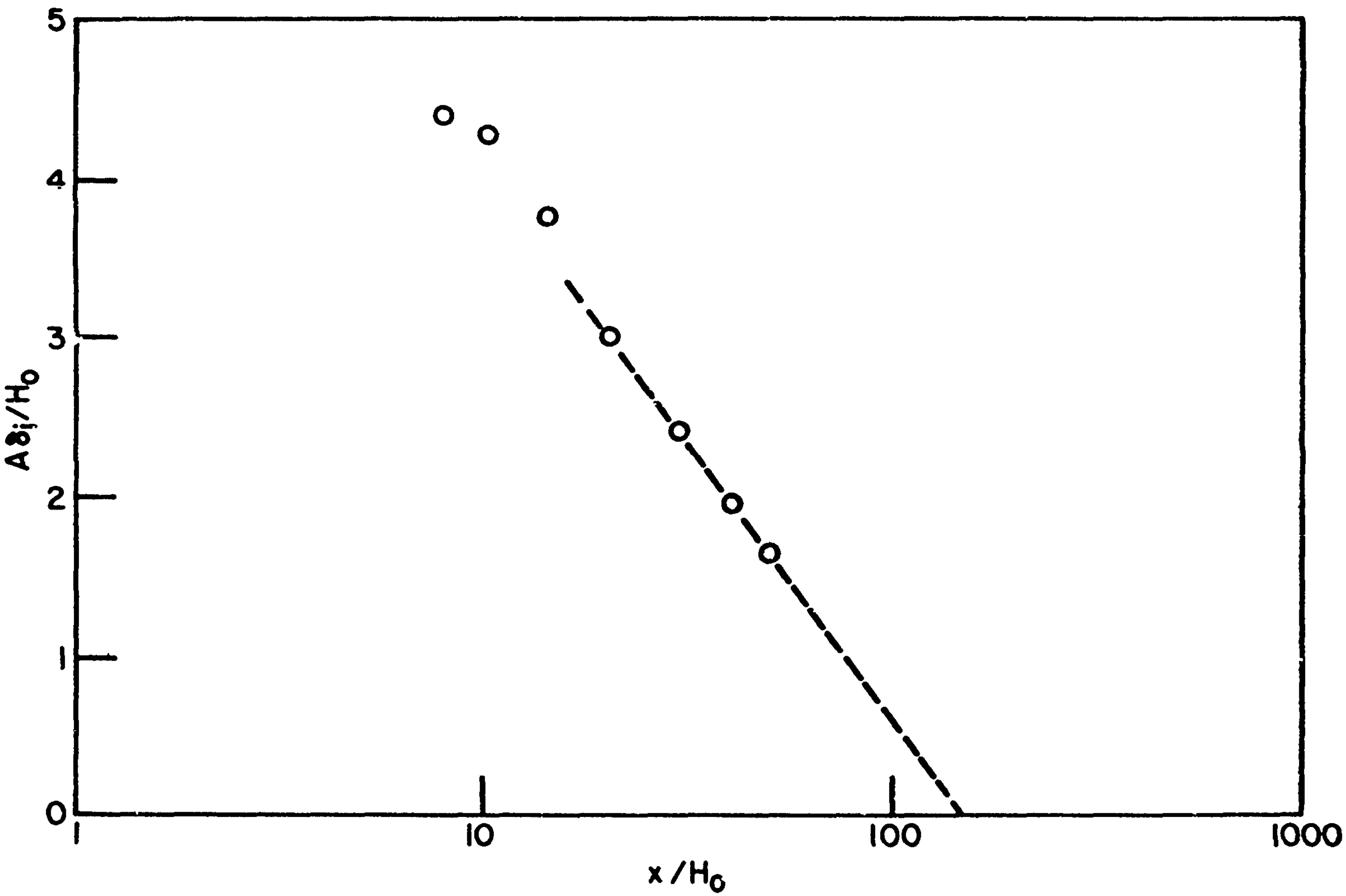

Fig. 9 - Normalized displacement thickness $A\delta_1/H_o$ as a function of nondimensional distance downstream for a momentumless wake, where A is a constant determined by the wake shape, based on data from Naudascher (1965).

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